BRIEF COMMUNICATIONS

A NOTE ON THE UNSTEADY BURNING RATE OF BALLISTITE POWDERS

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Let there be steady burning at t < 0, and let the pressure p begin to change at t = 0. It is required to study the behavior of the burning rate u(t) for small values of t.

STATEMENT OF THE PROBLEM AND PRESSURE RISE EFFECTS

In a coordinate system fixed relative to the powder, the equations for the temperature T and the concentration α have the form (W is the reaction rate):

$$\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial x^2}+Q\frac{W(T,\alpha)}{c\rho}, \tag{1}$$

$$\frac{\partial\alpha}{\partial t}=-W(T,\alpha), \tag{2}$$

$$t=0,\quad T=T_0(x),\quad \alpha=\alpha_0(x),$$
$$x=\infty,\quad T=T_1,\quad \alpha=1.$$

On the burning surface $x_s \quad \int_0^t u\,dt$ there is a given relation between the burning rate u, temperature gradient $-\varphi \quad \frac{\partial T}{\partial x}$ and pressure:

$$u=u(\varphi,p). \tag{3}$$

The considerations from which this relation was obtained in [1] remain valid when reactions in the powder are considered. We shall take the second condition on the surface in accordance with the ideas regarding the dispersion mechanism developed in [2]. It is proposed that the reaction generates gases, dissolved in the condensed phase, their concentration being $\sim(1-\alpha)$. There is a limit to the solubility at which the gas is liberated in the form of bubbles, which also creates dispersion. Therefore

$$1-\alpha_s\leqslant k\exp(L/RT_s). \tag{4}$$

Here L is the heat of solution. The dispersion condition is taken to have the sign of relation (4). When (4) is not an equality there is no dispersion, i.e., u = 0. This kind of situation may arise when the pressure rise is rapid enough, in fact, when

$$\frac{d\ln p}{dt}>\frac{W_s}{1-\alpha_s}+\frac{L}{RT_s^2}\left(a\frac{\partial^2 T}{\partial x^2}+\frac{QW_s}{c\rho}\right). \tag{5}$$

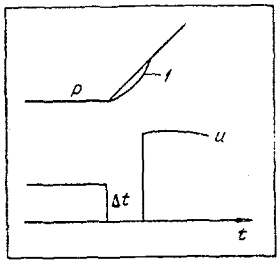

Fig. 1. Increase of burning rate and concentration of dissolved gas at the surface with sharp rise of pressure. 1) $\frac{1-\alpha_s}{k}\exp\left(-\frac{L}{RT_s}\right)$.

Relations (1), (2), and (4) have been used here. It appears that Eq. (5) may hold for a very short time, since $W_s/(1-\alpha_s)$ increases very quickly when there is no dispersion. When the equality of (4) is again true, accelerated burning begins (Fig. 1).

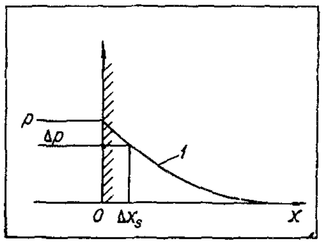

Fig. 2. Propagation of concentration of dissolved gas in the fuel. 1) $\frac{1-\alpha_0(x)}{k}\exp\left(-\frac{L}{RT_0(x)}\right)$.

Estimates show that Δt may be an order greater than the relaxation time of the gas stage. Therefore, when $0 < t < \Delta t$ a new boundary condition should be used instead of Eq. (3); for example, the heat flux should be found by solving the heat conduction equation in the gas in the absence of reactions with p = p(t).

SOLUTION OF (2) AND PRESSURE FALL EFFECTS

Let

$$W(T,\alpha)=\alpha f(T). \tag{6}$$

Then it follows from Eq. (2) that

$$\alpha=\alpha_0(x)\exp\left(-\int_0^t f[T(t,x)]\,dt\right). \tag{7}$$

In the steady case Eq. (2) takes the form

$$u\frac{d\alpha_0}{dx}=\alpha_0 f(T_0),$$

$$\alpha_0(x)=\exp\left(-\frac{1}{u}\int_x^\infty f(T_0)\,dx\right)=\alpha_{0s}\exp\left(\frac{1}{u}\int_0^x f(T_0)\,dx\right). \tag{8}$$

With the aid of Eqs. (4) and (7) certain conclusions may be drawn concerning u(t) when t → 0. For example, if the pressure drops discontinuously by Δp, the burning surface must jump to a location with values of α and T such that the equality (4) holds (Fig. 2), i.e., for Δx_s we have the equation

$$(1-\alpha_0(\Delta x_s))\exp(-L/RT_0(\Delta x_s))=p(+0). \tag{9}$$

A sharp increase in the number of dispersed particles has been observed in tests when there is a sudden drop in pressure.

EXPANSION OF (4) AS t → 0

In what follows we shall assume that from t = 0 onwards the pressure varies linearly, condition (5) not being satisfied:

$$\alpha_s=\alpha_0(x_s(t))\exp\left(-\int_0^t f[T(\sigma,x_s(t))]\,d\sigma\right)=$$

$$= \alpha_{0s} \exp \left(\frac{1}{u_0} \int\limits_0^{x_s(t)} f\,[T_0(x)]\,dx - \int\limits_0^{t} f\,[T(\sigma,\ x_s(t))]\,d\sigma \right) = \alpha_s(p,\ T_s).$$

Expanding the relation with an accuracy up to t, we obtain

$$\frac{u(+0)}{u_0} - 1 = \frac{1}{f(T_{s0})} \left. \frac{d \ln \alpha_s}{dt} \right|_{t=0} =$$

$$= \frac{k \exp (L/RT_{s0})}{f_{s0}\,\alpha_{s0}} \left(-\frac{dp}{dt} + p \frac{L}{RT_s^2} \frac{dT_s}{dt} \right)_{t=0}. \qquad (10)$$

Continuing the expansion, we obtain an asymptotic series for u. The variation of T_s and φ are taken into account by expanding the solution of Eq. (1) as in [3].

Thus, the problem of finding the unsteady burning rate by the Zel'dovich method may be brought into harmony with contemporary ideas regarding the mechanism of dispersion, if the ordinary boundary condition T_s = const is replaced by Eq. (4). It is then observed that during the relaxation time of the reaction zone in the condensed phase, effects may occur which contradict what is predicted by theory without allowance for reactions in the condensed phase.

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